

# Coherent and compatible information: a basis to information analysis of quantum systems

Boris A. Grishanin and Victor N. Zadkov

Faculty of Physics and International Laser Center, M. V. Lomonosov Moscow State University  
Moscow 119899, Russia

## ABSTRACT

Relevance of key quantum information measures for analysis of quantum systems is discussed. It is argued that possible ways of measuring quantum information are based on compatibility/incompatibility of the quantum states of a quantum system, resulting in the coherent information and introduced here the compatible information measures, respectively. A sketch of an information optimization of a quantum experimental setup is proposed.

**Keywords:** Quantum information, coherent information, compatible information

## 1. INTRODUCTION

The field of quantum information was born at the same time the basic laws of quantum physics had been established and since that time it plays an important role in physics. One could even say that quantum information theory was established prior the classical Shannon information theory. In favour of this, Bloch interpretation of the wave function or information meaning of the quantum collapse postulate could be mentioned.<sup>1</sup> Moreover, any quantum effect, i.e., essentially microscopic process of atom's spontaneous emission or macroscopic superconductivity transition, is associated with the corresponding process of quantum information transmission. Although importance of the quantum information concept was recognized long ago, not much attention has been paid to its practical importance until now, when modern experiments in quantum optics provide detailed control over quantum states of quantum systems. This allow us not only to think about quantum information as of an abstract concept, but apply it to real quantum systems and real experiments.

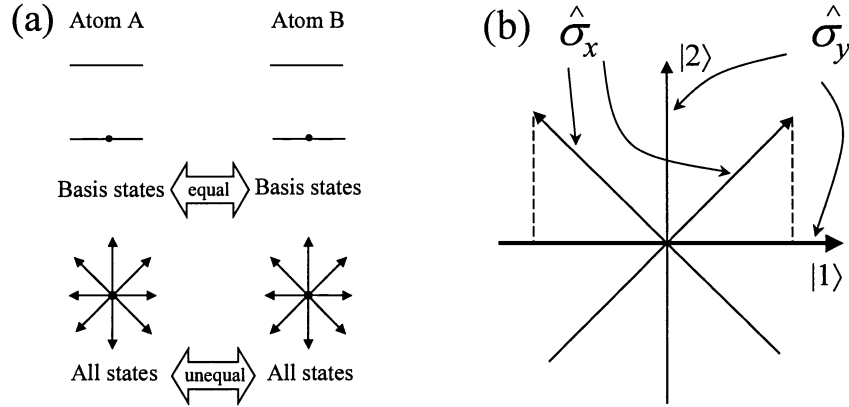
Sometimes it is expostulated that in physics one should necessarily deal with physical values, and if dealing only with physical states it is not physics but mathematics. Yet it is not true—whenever the states are specified as the states of a physical model, they provide physically meaning information. As an example, let us discuss an operator  $\hat{A}$  in Hilbert space  $H$  as a representation of a physical variable. Then, writing  $\hat{A}$  as a spectral decomposition  $\hat{A} = \sum \lambda_n |n\rangle \langle n|$  we represent it with two types of mathematical objects:  $\lambda_n$ , the possible physical values, and  $|n\rangle$ , the corresponding physical states. The latter contain the most general type of physical information on physical events regardless of the values  $\lambda_n$ .

The most general concept of classical information is the information theory introduced by Shannon.<sup>2,3</sup> This very elegant theory is based on the specific property of classical ensembles, which follows from the basic principles of quantum physics. This property is the *reproducibility* of classical events: statistically there is no difference either you have at input and output physically the same system or its informationally equivalent copies. The latter case is impossible in quantum world, which gives a rise to a discussion whether the Shannon approach can be applicable to the quantum systems or not.<sup>4-6</sup> As we will show, the traditional Shannon entropy and information measures can be successfully used for analysis of quantum systems, if correctly applied with clear understanding of the basic differences between the classical and quantum states ensembles.

Let us discuss, for example, two atoms in the same state (Fig. 1a). Term the “same” needs to be refined for the case of quantum systems, by contrast with its classical meaning. In the classical case, we take into account only two basis states of each atom. Then, we are free to suppose that either these basic states correspond to two different atoms or to one and the same atom. Important is that there is only one non-zero probability state in a combined system of two atoms—if a state of one of the two considered atoms is given, another atom has a non-zero probability state. In quantum case, two atoms have additional states with non-zero probability due to the internal quantum

---

Send correspondence to B.A.G.: grishan@comsim1.phys.msu.su



**Figure 1.** a) Equivalence of compatible basis-states ensembles and inequivalence of incompatible all-states ensembles of two two-level atoms. b) Vacuum fluctuations as a result of incompatibility: eigen states of  $\hat{\sigma}_x$  have equal non-zero probabilities  $p_{\pm} = 1/2$  at the eigen atomic state  $|1\rangle$ , thus providing nonzero fluctuations.

uncertainty (Fig. 1b). It is well known that this uncertainty results for a harmonic oscillator in vacuum fluctuation energy  $\hbar\omega/2$ . In our case of two-level atoms it takes the form of the non-zero values  $\hat{\sigma}_x^2 = \hat{\sigma}_y^2 = \hat{I}$ , where Pauli matrices  $\hat{\sigma}_{x,y}$  are treated as cosine and sine amplitudes of the atomic oscillator. The corresponding fluctuations are different for these two atoms, notwithstanding the latter are in the “same” state, which belongs to different atoms possessing individual internal incompatible ensembles of quantum states. Indeed, the average squared differences  $(\hat{\sigma}_x^A - \hat{\sigma}_x^B)^2$ ,  $(\hat{\sigma}_y^A - \hat{\sigma}_y^B)^2$  are both different from zero due to the non-commutativity of their operators with the population operators  $\hat{\sigma}_z$ , the latter yield certainly zero difference  $\hat{\sigma}_z^A - \hat{\sigma}_z^B$ .

What we can learn from the considered above example is that when ensembles of quantum system states are incompatible, i.e., non-orthogonal states of each atom (eigen states of the corresponding non-commuting operators) are involved, the states of two different atoms are always different with respect to all their ever coexisting internal quantum states allowed by the quantum uncertainty.

This statement can be expressed in a quantitative form as strict positivity of the average operator of the squared difference between the ortho-projectors onto the corresponding wave functions of the two atoms:

$$\hat{\varepsilon} = \int (|\alpha\rangle\langle\alpha| \otimes \hat{I}_B - \hat{I}_A \otimes |\alpha\rangle\langle\alpha|)^2 \frac{dV_\alpha}{D} = ||0\rangle\rangle\langle\langle 0|| + \frac{1}{3} \sum_{k=1}^3 ||k\rangle\rangle\langle\langle k|| \geq \frac{1}{3},$$

where integration is made over the Bloch sphere of the states  $|\alpha\rangle$  with the volume differential  $dV_\alpha = \sin\vartheta d\vartheta d\varphi / (2\pi)$  and the total volume  $V_\alpha = D = 2$ . This bipartite operator has two eigen subspaces composed of a singlet and triplet Bell states  $||k\rangle\rangle$ , corresponding to the eigen squared difference values  $\varepsilon_k = 1, 1/3$ , the singlet one being three times bigger.

At this point, one can conclude that the key difference between classical and quantum information lies in *compatibility* or *incompatibility* of the states associated with the information of interest. The one-time states of different systems are always compatible. Therefore, they cannot copy one another if states of each system include internally incompatible states. Conversely, two-time states of the deterministically transformed system are always incompatible. Two-time states of different systems can be either compatible or not.

In this paper, we will classify quantum information in connection with the compatibility property described above. In this vein, we can distinguish four main types of information listed below:

- *Classical information*—all the states are compatible and in original form of information theory quantum systems are not discussed.<sup>2,3</sup> Note that classical information can be well transmitted through the quantum channels and also can be of interest in Quantum Physics.

- *Semiclassical information*—all the input information is given by classical states  $\lambda$  and the output states include internal incompatibility in the form of all states of a Hilbert space  $H$ , which are automatically compatible with the input states. The quantum channel is generally described via a classical parameter dependent on the ensemble of mixed states  $\hat{\rho}_\lambda$ .<sup>7,8</sup>
- *Coherent information*—both input and output are spaces composed of internally incompatible states, plus these spaces are also incompatible and connected via a channel superoperator  $\mathcal{N}$  transforming the input density matrix into the output one:  $\hat{\rho}_B = \mathcal{N}\hat{\rho}_A$ .<sup>9,10</sup> It is a “flow” of quantum incompatibility from one system to another.
- *Compatible information*—both input and output are composed of internally incompatible states, which are mutually compatible.

While three first types of information were thoroughly discussed in the literature,<sup>3,7,9</sup> including the recently introduced coherent information measure, the compatible information is introduced here for the first time. This new type of quantum information is defined for a compound bipartite quantum system with the compatible input and output, which include internal quantum incompatibility.

In our view, the coherent and compatible information exhaust all possible qualitatively different types of information in quantum channels. Presented in the paper feasibility analysis of using these two measures of information for information analysis of real experimental schemes shows that only compatible information turns to be suitable for information effectiveness analysis of an experimental scheme (in the following we will simply call an experimental scheme an “experimental setup”).

## 2. COHERENT INFORMATION

### 2.1. Physical meaning of coherent information

The coherent information quantitatively represents an amount of incompatible information, which is transferred from one space to another. A case of one and the same space can be considered, as well. A trivial case of the coherent information exchange is a dynamic evolution represented with the unitary time evolution operator  $U$ ,  $\hat{\rho}_B = U\hat{\rho}_AU^{-1}$ . Then, all pure states  $\psi$  allowed by the initial density matrix  $\hat{\rho}_A$  are transformed with no distortion, and the transmitted coherent information coincides with its initial amount. The latter is measured, by definition, with the von Neumann entropy, which reads

$$I_c = S[\hat{\rho}_B] = S[\hat{\rho}_A] = -\text{Tr} \hat{\rho}_A \log \hat{\rho}_A. \quad (1)$$

This definition yet demands additional justifying in terms revealing an operational meaning of the density matrix, which is given in a self-consistent quantum theory as a result of averaging of a pure state in a compound system over the auxiliary variables. Then, Eq. (1) describes an entanglement of the input system  $A$  with a *reference* system  $R$ , which corresponds to a proper pure state  $|\Psi_{AR}\rangle$ ,  $\langle \Psi_{AR}| = \hat{\rho}_A$  of a combined  $A+R$  system. Thus quantitative measuring of the coherent information is done in terms of the mutually compatible states of two different systems,  $A$  and  $R$ , while information transfers from input  $A$  to the output  $B$ , which differs from  $A$  here only with a unitary transformation.

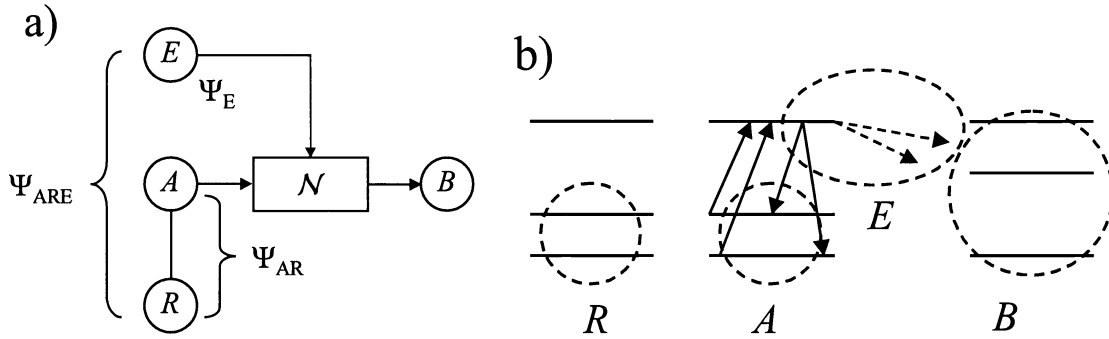
To complete the general structure of the information system, an information channel  $\mathcal{N}$  with the attached noisy *environment*  $E$  should be added (Fig. 2a).<sup>11</sup>

The definition of the coherent information for a general type of channel reads as<sup>11</sup>

$$I_c = S[\hat{\rho}_B] - S[(\mathcal{N} \otimes \mathcal{I}) |\Psi_{AR}\rangle \langle \Psi_{AR}|], \quad (2)$$

where  $\mathcal{I}$  is the identical superoperator applied to the variables of the reference system. The second term is the *entropy exchange*, which is non-zero due to the exchange between the subsystems  $A+R$  and  $E$ , which is when  $\mathcal{N} \neq \mathcal{I}$ . Channel superoperator  $\mathcal{N}$  transforms the states of input  $A$  according to the equation

$$\hat{\rho}_B = \mathcal{N}\hat{\rho}_A = \text{Tr}_R(\mathcal{N} \otimes \mathcal{I}) |\Psi_{AR}\rangle \langle \Psi_{AR}| \quad (3)$$



**Figure 2.** a) The most general scheme of quantum information system, composed of input  $A$ , reference system  $R$ , channel  $\mathcal{N}$  with noisy environment  $E$ , and output  $B$ . b) An example of physical implementation of a quantum information system: an input  $A$  and a reference system  $R$  are the ground states of two entangled atomic  $\Lambda$ -systems, information channel  $\mathcal{N}$  is provided with the laser excitation of an input system to the radiative upper level, the two photon field states corresponding to the emitted photons together with the vacuum state provide an output  $B$ , and all other field states together with the excited atomic state form the environment  $E$ .

to the states of output  $B$ , which is again compatible with the reference system  $R$  because of no entanglement between them at this transformation. A physical meaning of Eq. (2) is switched then from an incompatibility flow to a specific measure for a *preserved* entanglement between the *compatible* systems  $R$  and  $B$ , which is left after transmission through the channel. In a general case, output  $B$  may be physically different from  $A$  and even represented with a Hilbert space of different structure,  $H_B \neq H_A$ ,<sup>12,13</sup> as shown for a specific example of a physical information system in Fig. 2b.

Now we will try to answer a question how the coherent information measure can be used in physics? Quantum theory is usually applied to the calculation of some average values  $\langle \hat{A} \rangle = \sum \lambda_n \langle |n\rangle \langle n| \rangle$ , where  $\lambda_n$  and  $|n\rangle$  denote the eigen values and eigen vectors of an operator  $\hat{A}$ . This expansion represents averaging of physical variables in terms of probabilities  $P_n^A = \langle |n\rangle \langle n| \rangle$  of quantum states  $|n\rangle$ . As far as there is an innumerable set of all possible variables and it is much richer than the set of all quantum states, description of the correspondences between the physical states, apart of physical values, provides a more general information on the physical correspondences the most economical way. Laws for coherent information exchange follow the most basic laws of quantum physics, as they show the most general features of interaction between two systems of interest chosen as input and output and connected with a one-to-one transformation of the input states. In fact, the dependencies of the coherent information on the system parameters are even more basic than those of specific physical values.

Let us consider, for example, a Dicke problem for which an information exchange shows the same oscillation type of dynamics as the energy exchange between the two atoms, assisted with the radiation damping.<sup>12</sup> This oscillatory evolution is characteristic not only for the energy, but also for many other variables. Therefore, there is a point in considering evolution of the coherent information instead of working with many other variables. One should also keep in mind the physical meaning of the coherent information as a preserved entanglement. The latter, in its turn, is a characteristic of an internal incompatibility exchange between the mutually compatible sets of states for the reference and input systems,  $H_R$  and  $H_A$ . Among other types of quantum information the coherent information distinguishes between two types of information, corresponding to the exchange via classical information and quantum entanglement. The coherent information is nonzero only for the latter case. Thus, it is adequate to discuss how well the given information transmission channel preserves the capability of using the output as an equivalent of the input to realize a task, when quantum properties of a signal are essential. This problem received much attention in the literature (see Ref. 14 and references therein).

One can also be interested in applying the coherent information concept to an analysis of a specific model of a quantum channel. One of the examples is discussed in Sec. 2.3.

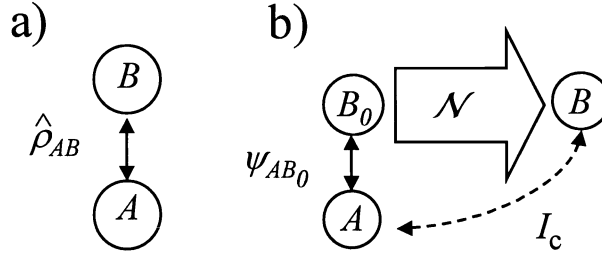
## 2.2. One-time coherent information

A first step towards information characterization of a two-side quantum channel could be undertaken by formal quantum generalization of the classical Shannon mutual information  $I = S_A + S_B - S_{AB}$ :

$$I = S[\hat{\rho}_A] + S[\hat{\rho}_B] - S[\hat{\rho}_{AB}], \quad (4)$$

which is valid if the joint density matrix  $\hat{\rho}_{AB}$  is given and treated as a strict analogue of classical joint probability distribution  $P_{AB}$ .<sup>15</sup> Evidently, to apply this formula to quantum systems we should suppose that  $A$  and  $B$  states are mutually compatible, which is valid for the one-time states of the corresponding physical systems, unless they belong to the same system, both as input and output. Note at this point that physical meaning of  $I$  still remains unclear.<sup>16,17</sup> It could be clarified by taking into account striking difference between the classical and quantum information channels. Generally, as it follows from Eq. (3), quantum input and output are incompatible, being taken for a single system at two time instants. Thus,  $A$  and  $B$  cannot be treated as input and output, and their further specification must be made for the quantum case.

Let us then specify  $A$  as the reference system and  $B$  as the output for a given joint density matrix  $\hat{\rho}_{AB}$  as it is shown in Fig. 3. The input  $B_0$  and the channel  $\mathcal{N}$  are not introduced explicitly but through their action, resulting in the given density matrix  $\hat{\rho}_{AB}$ .



**Figure 3.** Reconstruction of the quantum information system corresponding to the given joint density matrix  $\hat{\rho}_{AB}$ : mathematical description of a channel providing one-time coherent information (a) and correspondence with the Schumacher's treatment<sup>11</sup> (b).

The pure state  $\Psi_{AB_0}$  of the input–reference system and the channel superoperator  $\mathcal{N}$  should obey the equation

$$\hat{\rho}_{AB} = (\mathcal{I} \otimes \mathcal{N}) |\Psi_{AB_0}\rangle \langle \Psi_{AB_0}|. \quad (5)$$

This automatically provides the coincidence of the partial density matrix of the reference state

$$\hat{\rho}_A = \text{Tr}_{B_0} |\Psi_{AB_0}\rangle \langle \Psi_{AB_0}|$$

with the partial density matrix  $\hat{\rho}_A = \text{Tr}_B \hat{\rho}_{AB}$  calculated by averaging of the given  $A+B$  state, as far as trace over  $B_0$  of Eq. (5) is invariant on  $\mathcal{N}$ .

Then, the corresponding *one-time* coherent information can be defined as

$$I_c = S[\hat{\rho}_B] - S[\hat{\rho}_{AB}], \quad (6)$$

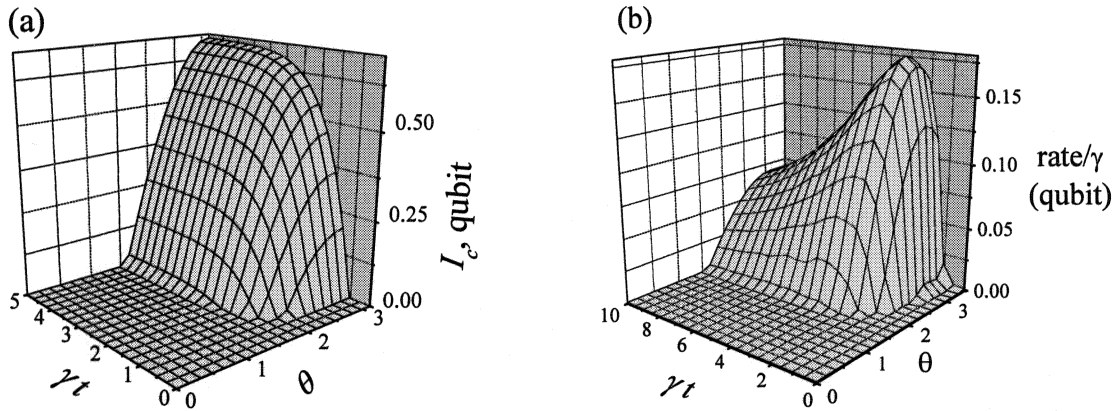
which by contrast with the quantity (4) lacks the term  $S[\hat{\rho}_A]$ . Term “one-time” here may not have in general case a strict meaning, because any two compatible quantum systems  $A$  and  $B$ , even related to different time instants, can be treated as related to the one time instant after the corresponding transformation of states.

Additional property of one-time coherent information is that definition (6) lacks symmetry by contrast with (4). Moreover, the coherent information can be negative. The latter is evident for the density matrices  $\hat{\rho}_{AB}$  corresponding to the purely classical information exchange via orthogonal bases,  $\hat{\rho}_{AB} = \sum P_{ij} |i\rangle |j\rangle \langle j| \langle i|$ . Then, the entropies reduce to the classical entropies  $S[\hat{\rho}_{AB}] = S_{AB} = -\sum P_{ij} \log P_{ij}$ ,  $S[\hat{\rho}_B] = S_B = -\sum P_j \log P_j$  and  $S_{AB} > S_B$ . Negative value of the coherent information means that the entropy exchange prevails information transmission, so it is reasonable to set  $I_c = 0$  in this case.

### 2.3. Coherent information exchange rate in the $\Lambda$ -system

Information system presented in Fig. 2b plays a special role in new applications based on nonclassical properties of quantum information, e.g., quantum cryptography and quantum computations. Key elements in such applications are atomic  $\Lambda$ -systems, which thought to be promising elements (qubits) to store quantum information and are convenient to manipulate with the help of laser radiation.<sup>14,18</sup> For our system (Fig. 2b) treating second  $\Lambda$ -system as a reference system has a reasonable justification, as the entanglement of two corresponding qubits has a clear physical meaning of the initially provided quantum information. Discussion of the radiation channel is also interesting, because the transformation of the initial qubit into the photon field enables a wide choice of subsequent transformations. A particular question that can be raised here is how rapidly could the information be recycled after a single use of a qubit-photon field channel?

Details of the calculations of the coherent information exchange for this channel are given in Ref. 13. The dependence of the coherent information on time and laser field action angle for a symmetric  $\Lambda$ -system is shown in Fig. 4a for a maximum entropy qubit state  $\hat{\rho}_A = \hat{I}/2$ , when information does not depend on the individual field intensities of the two applied laser fields.



**Figure 4.** a) The coherent information in a symmetric  $\Lambda$ -system as a function of dimensionless time  $\gamma t$  and action angle  $\theta = \Omega \tau_p$  for the maximum entropy input state;  $\gamma$  is the decay rate,  $\Omega$  is the effective Rabi frequency and  $\tau_p$  is the exciting pulse duration.<sup>12</sup> b) Dependence of the information rate on the cycle duration  $t$  and action angle  $\theta = \Omega \tau_p$ .

It can be easily seen from Fig. 4a that there is an optimum value for the information rate  $R = I_c/t$ ,  $t = \tau_c$ , if we introduce a periodic use of the information channel with a cycle duration  $\tau_c$ , so that after each cycle the initial state is instantaneously renewed. The calculation results for the rate  $R$  for a symmetric  $\Lambda$ -system with the partial decay rates  $\gamma_1 = \gamma_2 = \gamma$  are shown in Fig. 4b.<sup>19</sup> The total optimum rate is  $R_0 = 0.178\gamma$ . Thus, the process of atom-photon field information exchange sets the corresponding rate limit on using the coherent information stored in the  $\Lambda$ -systems. The order of its magnitude is given by the decay rate of the excited state, while an exact value depends on the partial decay rates  $\gamma_{1,2}$  of the  $\Lambda$ -system transitions. At the limit of a two-level radiative system,  $\gamma_1 = 0$  or  $\gamma_2 = 0$ , the optimum rate is equal to  $0.316\gamma$ .

### 3. COMPATIBLE INFORMATION

For one-time average values, one can restrict representation of quantum internal incompatibility in an equivalent form of classical probability distribution on the quantum states of interest. Then, for the probability measure

$$P(d\alpha) = \langle \alpha | \hat{\rho}_A | \alpha \rangle dV_\alpha \quad (7)$$

on the space of all quantum states the average value of an operator  $\hat{A} = \sum \lambda_n |n\rangle \langle n|$  can be written as  $\langle \hat{A} \rangle = \sum \lambda_n dP/dV_\alpha(\alpha_n)$ , where  $|\alpha_n\rangle = |n\rangle$ . Here  $dV_\alpha$  is the volume differential in the space of physically different states of

the  $D$ -dimensional Hilbert space  $H_A$  ( $\int dV_\alpha = D$ ), which, for example, for a qubit system with  $D = 2$  is the Bloch sphere (see Sec. 2.1). Eq. (7) is an average of the projective measure

$$\hat{E}(d\alpha) = |\alpha\rangle\langle\alpha| dV_\alpha, \quad (8)$$

which is a specific case of non-orthogonal decomposition of unit,<sup>20</sup> or positive operator-valued measure (POVM).<sup>21</sup> POVMs represent some physical measurement procedures made in a compound space  $H_A \otimes H_a$  with an appropriate additional space  $H_a$  and joint density matrix  $\hat{\rho}_A \otimes \hat{\rho}_a$ , which gives no additional information about  $A$  beyond the information given by  $\hat{\rho}_A$ .

Let us assume that two Hilbert spaces,  $H_A$  and  $H_B$ , of the corresponding quantum systems  $A$  and  $B$  and the joint density matrix  $\hat{\rho}_{AB}$  in  $H_A \otimes H_B$  are given. Specifically, they can correspond to the subsystems of a compound system  $A+B$ , given at the same time instant  $t$ , or be defined as input and output of an abstract quantum channel of a real physical system. Described above subsystems  $A$  and  $B$  are compatible. Therefore, a joint measurement represented with the two POVMs as  $\hat{E}_A \otimes \hat{E}_B$  gives no extra correlations between output and input measurements and the respective joint input–output probability distribution takes the form:

$$P(d\alpha, d\beta) = \text{Tr} [\hat{E}_A(d\alpha) \otimes \hat{E}_B(d\beta)] \hat{\rho}_{AB}. \quad (9)$$

The corresponding Shannon information  $I = S[P(d\alpha)] + S[P(d\beta)] - S[P(d\alpha, d\beta)]$  defines then the *compatible* information measure.<sup>22</sup>

The physical meaning of the compatible information depends on the specific choice of the measurement and represents the quantum information on input obtainable from the output via the POVMs, which select the information of interest in the classical form of the corresponding  $\alpha$  and  $\beta$  variables, the information carriers.

Let us consider the case when  $\alpha$  and  $\beta$  enumerate all the quantum states of  $H_A$  and  $H_B$ , in accordance with Eq. (8). In this case, compatible information is distributed over all quantum states and associated with the internal quantum uncertainty, which is taken into account in the distribution (7). Specifically, quantum correlations due to the possible entanglement between  $A$  and  $B$  are taken into account in the joint probability (9). Moreover, the compatible information in this case yields the *operational invariance* property,<sup>23</sup> which is when all the non-commuting physical variables are taken into account equivalently. Such classical representation of the quantum information can be associated with the representations of quantum mechanics in terms of classical variables.<sup>24</sup>

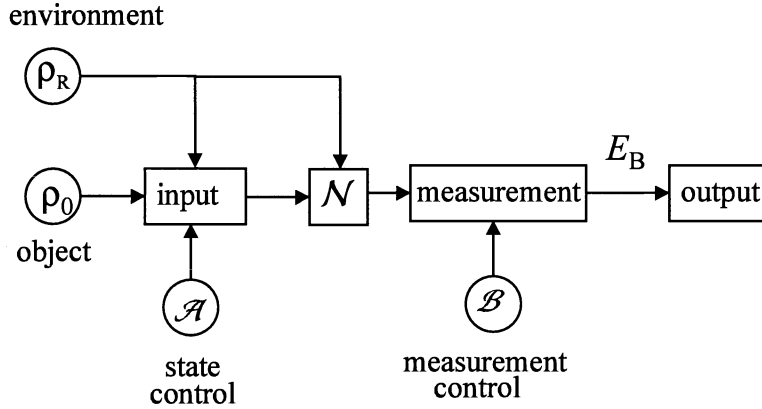
#### 4. AMOUNT OF INFORMATION ATTAINABLE BY AN EXPERIMENTAL SETUP

Our previous discussion of the generalized measurements encourages us to introduce in this section a likelihood mathematical concept of information attainable by an experimental setup, which certainly is one of the key goals of the Quantum Information Theory. It is difficult to define the information model corresponding to the experimental setup under consideration in general form. Therefore, one has first to specify the input and output information of interest (which is actually the most difficult point here). We propose here a solution illustrated by the block scheme shown in Fig. 5.

This block scheme corresponds to a typical mathematical structure of a density matrix of a complex system including two transformations,  $\mathcal{A}$  and  $\mathcal{B}$ , representing control and measurement interactions, correspondingly:

$$\hat{\rho}_{\text{out}} = \mathcal{B}\mathcal{N}\mathcal{A}\hat{\rho}_{\text{in}}. \quad (10)$$

Here  $\hat{\rho}_{\text{in}}$  and  $\hat{\rho}_{\text{out}}$  are the initial and final density matrices for the degrees of freedom, chosen in a mathematical model of the experimental setup. Superoperators  $\mathcal{A}$ ,  $\mathcal{B}$ , and  $\mathcal{N}$  are associated with the preparation of the information, the measurement, and the transmission of the information to the output, correspondingly. This markovian-type structure is not the most general one—for simplicity we assume that the reservoirs corresponding to each transformation are independent and their density matrices can be separated from  $\hat{\rho}_{\text{in}}$ . Only under this simplification we can get a separated combination of the three superoperators and the input density matrix and, as a result, get a relatively simple mathematical representation of the information structure in terms of the corresponding decompositions of  $\mathcal{A}$  and  $\mathcal{B}$ . Still, we have to keep in mind that a proper generalization of Eq. (10) may be necessary in a general case.



**Figure 5.** Information structure of a quantum experimental setup. An object accompanied with the noise environment undergoes the state control interactions, produces the input information ensemble, depending on either the object dynamical parameters or quantum states of interest. Then, after the channel superoperator transformation  $\mathcal{N}$  the output information is measured.  $\mathcal{A}$  and  $\mathcal{B}$  denote transformations provided with the controlling interactions,  $E_B$  stays for the measurement procedure in the form of the corresponding POVM.

Preparation of the information always involves some interactions, resulting in the corresponding transformations, which are unitary only if all the involved degrees of freedom are taken into account. We have to include also interaction with the reservoir represented with a non-unitary superoperator. We will discuss here the recipes for two possible choices of a physical information of interest:

- (i) the system dynamic parameters  $a$ ,
- (ii) the system dynamic states  $|a\rangle$ .

For the choice (i), the required information goal can be achieved with the use of the dynamical evolution operators  $U_A(a)$ , which in its turn may depend on the controlling parameters  $c$ . *A priori* information on  $a$  is included in a proper chosen probability measure  $\mu(da)$ . Corresponding superoperator  $\mathcal{A}$  is then can be written as  $\mathcal{A} = \int \mathcal{A}_a \mu(da)$  with

$$\mathcal{A}_a = \langle U_A(a) \odot U_A^{-1}(a) \rangle_E, \quad (11)$$

where symbol  $\odot$  denotes the place to substitute with the transformed density matrix and brackets denote averaging over the noise environment.

For the choice (ii), the required information goal can be achieved with the use of the measurement superoperator transformation composed of superoperators

$$\mathcal{A}_a = \langle |a\rangle \langle a| \odot |a\rangle \langle a| \rangle_E. \quad (12)$$

The corresponding sum  $\mathcal{A} = \sum \mathcal{A}_a$  is the measurement superoperator represented with an averaged standard decomposition  $\sum_i \hat{A}_i \odot \hat{A}_i^\dagger$  of the completely positive trace-preserving superoperator<sup>25</sup> with a properly specified operators  $\hat{A}_i = \hat{A}_i^\dagger \rightarrow |a\rangle \langle a|$ . Keeping in mind that  $a$  can represent a continuous variable, we have to use a generalized representation  $\mathcal{A} = \int \mathcal{A}_a \mu(da)$  in the integration form with a proper measure  $\mu(da)$ , providing a corresponding decomposition of unit (POVM)  $\int |a\rangle \langle a| \mu(da) = \hat{I}$ .

In most general form, the superoperator sets (11), (12) are represented with an arbitrary *positive superoperator measure* (PSM)  $\mathcal{A}(da) = \mathcal{A}_a \mu(da)$ , which is a decomposition of a completely positive trace-preserving superoperator. PSM satisfies the conditions of complete positivity,  $\mathcal{A}(da)\hat{\rho} \geq 0$ , and normalization,  $\text{Tr} \int \mathcal{A}(da)\hat{\rho} = 1$ . The latter can be expressed in an equivalent form of preservation of the unit operator  $\int \mathcal{A}^*(da)\hat{I} = \hat{I}$  by the conjugate PSM  $\mathcal{A}^*$ .

It is worth to discuss here a special case when the POVMs are represented by Eq. (8) with all the states of the Hilbert spaces  $H_A$  and  $H_B$  corresponding to  $\mathcal{A}$  and  $\mathcal{B}$ , again. This definition of the POVMs restricts the information



attainable by an experimental setup due to the basic physical limitations underlying the chosen mechanism of obtaining quantum information. The latter is represented here in a “solid” classical form enabling its copying and free use. This property may as well be assigned by default to the meaning of the term “information”, by contrast to the opposing meaning of the coherent information discussed in Secs. 2.1–2.3.

Repeating the above argumentation for the measurement superoperator  $\mathcal{B} = \int \mathcal{B}(db) = \int \mathcal{B}_b \nu(db)$  with  $\mathcal{B}_b$  in the form of Eq. (12), we can implement the input and output information in the form of classical variables  $a$  and  $b$  for both choices, (i) and (ii), of the information of interest. The corresponding joint probability distribution is then given by

$$P(da, db) = \text{Tr} \mathcal{B}(db) \mathcal{N} \mathcal{A}(da) \hat{\rho}_{\text{in}}. \quad (13)$$

This distribution is always positive and normalized to 1. It gives an experimenter the statistical correspondence between the states of interest and output information attainable by the experimental setup. The corresponding *information capacity* of the setup can be expressed in the quantitative form as the responding Shannon information, which then can be used for optimization of the setup parameters.

It is important to note that mutual compatibility of the  $|a\rangle$  and  $|b\rangle$  states for (i) choice is not declared here and, in general case, the states can correspond to the non-commuting projectors. In a trivial extreme, they could be the same states and all the information is sent with zero error probability. If the states belong to the different physical subsystems, they may carry on quantum correlations due to the corresponding structure of the channel superoperator  $\mathcal{N}$ . A simplest example could be given by  $\mathcal{N} = U_{AB} \odot U_{AB}^{-1}$  with  $U_{AB}$  being the entangling unitary transformation.

The control parameters  $c$  may be either fixed or be set of used values  $c \in \mathbb{C}$ . For their optimization one can use the Shannon information measure. The unknown probability distribution  $\mu(da)$  of the dynamical parameters  $a$  for the case (i) can be calculated in terms of the classical decision theory<sup>26</sup> and no quantum mechanics is necessary. As for the specification of the action  $\mathcal{B}_b$  of the measurement system in the form (11), it may be generalized in the form of a general type PSM. Two PSMs  $\mathcal{A}(da)$  and  $\mathcal{B}(db)$  cover a wide range of state control and measurement systems implemented into the model of the experimental setup.

## 5. CONCLUSIONS

In the paper we classified the quantum information into the *classical*, *semiclassical*, *coherent*, and *compatible* information based on the compatibility property. This list exhausts all basically different types of quantum information.

Physical meaning of the coherent information is an amount of the internal incompatibility exchanged between two systems and measured as an entanglement *preserved* between the output and the reference system. Introduced here *one-time coherent information* sets a correct correspondence between the Schumacher’s and modified Stratonovich’s approaches. We calculated the coherent information exchange *rate* of a  $\Lambda$ -system via photon field that does not exceed  $0.178\gamma$  for a symmetric  $\Lambda$ -system and  $0.316\gamma$ , otherwise.

We introduce here for the first time the *compatible information*, which is an adequate characteristic of the quantum information exchange between compatible systems. The compatible information can be expressed in terms of classical information despite internal incompatibility, by contrast with the coherent information, which is basically irreducible to the classical terms.

It is shown that internal compatibility of the input and output quantum information seems an adequate restriction for a physical information in an experimental setup. It makes possible quantitative characterization of the available *information capacity* of the experimental setup. Then, information exchange between the subsystem, preparing information, and the measuring device is formulated as a probabilistic correspondence between the classical variables determining the corresponding dynamical evolution and the measured output values. A general mathematical representation of the information generation and its readout is presented in the form of two PSMs. This representation of physical information exchange in an experimental setup seems to be promising in direct application of Quantum Information Theory to the demands of experimental physics.

## ACKNOWLEDGMENTS

This work was partially supported by RFBR grant no. 01–02–16311, the State Science-Technical Programs of the Russian Federation “Fundamental Metrology” and “Nano-technology”, and INTAS grant no. 00–479.

## REFERENCES

1. A. Sudbery, *Quantum Mechanics and the Particles of Nature*, Cambridge Univ. Press, New York, 1986.
2. C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication*, University of Illinois Press, Urbana, 1949.
3. R. G. Gallager, *Information Theory and Reliable Communication*, John Wiley and Sons, New York, 1968.
4. Č. Brukner, A. Zeilinger, LANL e-print quant-ph/0006087.
5. M. J. W. Hall, LANL e-print quant-ph/0007116.
6. Č. Brukner, A. Zeilinger, LANL e-print quant-ph/0008091.
7. A. S. Holevo, *Probl. Inf. Trans.* **9**, 177 (1973).
8. M. J. W. Hall, *Phys. Rev. A* **55**, 100 (1997).
9. B. Schumacher and M. A. Nielsen, *Phys. Rev. A* **54**, 2629 (1996).
10. S. Lloyd, *Phys. Rev. A* **55**, 1613 (1997).
11. H. Barnum, B. W. Schumacher, and M. A. Nielsen, *Phys. Rev. A* **57**, 4153 (1998).
12. B. A. Grishanin and V. N. Zadkov, *Phys. Rev. A* **62**, 032303 (2000).
13. B. A. Grishanin and V. N. Zadkov, *Laser Physics* **10**, No. 6, 1280 (2000).
14. *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, D. Bouwmeester, A. Ekert, A. Zeilinger, Eds., Springer, Berlin, 2000.
15. R. L. Stratonovich, *Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika* **8**, 116 (1965).
16. G. Lindblad, *Lect. Notes Phys.* **378**, *Quantum Aspects of Optical Communication*, C. Benjaballah, O. Hirota, and S. Reynaud, Eds., 71 (1991).
17. A. S. Holevo, LANL e-print quant-ph/9809022.
18. I. V. Bargatin, B. A. Grishanin, and V. N. Zadkov, *Physics–Uspekhi* **171**, 625 (2001). (In Russian.)
19. D. Bokarev, private communication (2001).
20. B. A. Grishanin, *Tekhnicheskaya Kibernetika*, **11** (5), 127 (1973).
21. J. Preskill, *Lecture notes on Physics 229: Quantum information and computation*, available on Internet at <http://www.theory.caltech.edu/people/preskill/ph229/>.
22. B. A. Grishanin and V. N. Zadkov, *Laser Physics* (in press).
23. C. Brukner and A. Zeilinger, *Phys. Rev. Lett.* **83**, 3354 (1999).
24. R. J. Glauber, “Optical coherence and photon statistics”, In: *Quantum Optics and Electronics*, C. DeWitt, A. Blandin, and C. Cohen-Tannoudji, Eds, Gordon&Breach, New York, 1965.
25. K. Kraus, *States, Effects and Operations*, Springer Verlag, Berlin, 1983.
26. A. Wald, *Statistical Decision Functions*, Wiley, New York, 1950.