An Insight into Fundamental Problems of Quantum Information in Physics

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ABSTRACT

The main quantum information measures are discussed with respect to their relation to physics. It is argued that the basic term to choose between the possible ways to measure quantum information is compatibility/incompatibility of the quantum states, resulting in coherent information and here suggested compatible information measures. A sketch of information optimization of a quantum experimental setup is proposed.

Keywords: Quantum information, coherent information, compatible information

1. INTRODUCTION

Quantum information had started its play in physics right since its basic laws been established as quantum ones. To make a paradox, one even may say that quantum information theory had been established earlier than classical Shannon theory. To support this thesis just Bloch interpretation of the wave function could be mentioned, contributed with information meaning of quantum collapse postulate.[1] Moreover, any quantum effect, say, essentially microscopic process of atom's spontaneous emission or macroscopic superconductivity transition, is involved in the corresponding process of quantum information transmission. Yet the idea of quantum information as a practically applicable physical concept had not gained its present meaning until a proper experimental support been provided and control of quantum states become available. As far as now quantum information is capable to manipulate, it is worth discussion in as general terms as possible.

There two opposite interpretations of the correspondence between Quantum Mechanics and Quantum Information could be distinguished. First, there is an attempt to reconstruct the foundations of Quantum Mechanics on the base of existing ideas of the Quantum Information Theory.[2] And the other one is an attempt to modify the Quantum Information Theory on the base of existing axioms of Quantum Mechanics.[3,4] The latter seems more appropriate for the purposes of this paper, as far as by now Quantum Information Theory does not seem to be a completed one.

Sometimes it is expostulated that physics necessarily deals with physical values, and what deals only with physical states is not physics but mathematics. Yet is not true. Whenever the states are specified as the ones of a physically meaning model, they provide physically meaning information. For example, let we discuss an operator $\hat{A}$ in a Hilbert space H as a representation of a physical variable. Then spectral decomposition $\hat{A} = \sum \lambda_n |n\rangle \langle n|$ provides its splitting in two types of mathematical objects, λ_n the possible physical values and $|n\rangle$ the corresponding physical states. The latter contain the most general type of physical information on physical events regardless the values λ_n.

The most general approach to classical information is information theory introduced by Shannon.[5,6] Yet this very elegant theory is based on the specific property of classical ensembles which is exiled from the basic principles of quantum physics. This property is reproducibility of classical events: statistically it is no difference either you have at input and output physically one and the same system or its informationally equivalent copies. The latter case is impossible in quantum world. This difference is illustrated with Fig. 1.

This even gives rise to discussion whether the Shannon approach is anyway applicable to quantum systems or not.[7–9] Here we are with the traditional viewpoint, that Shannon entropy and information measures yet may be successfully used, while applied an appropriate way and with clear understanding of the basic differences between the classical and quantum state ensembles.

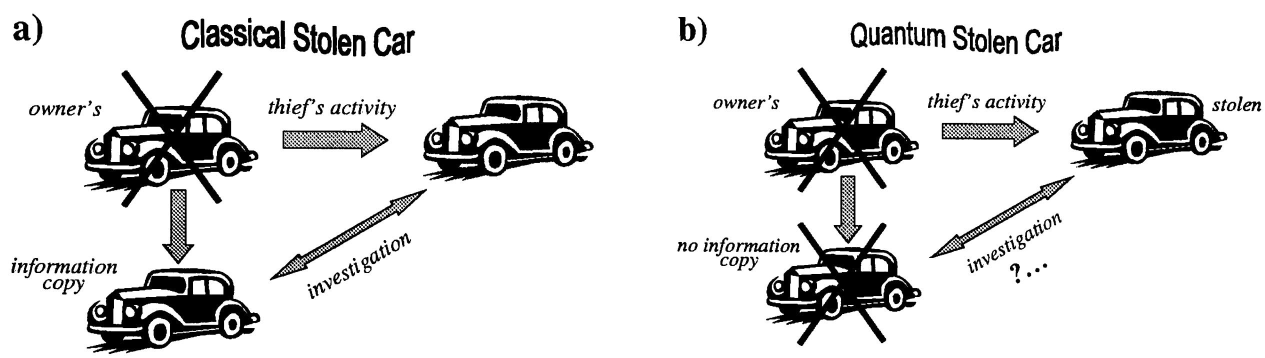

Figure 1. The main information difference between classical and quantum world: a) it is easy to estimate that a specific car is the one which had been stolen, basing on an exact information copy of the original car; b) if the car were an essentially quantum object it would be a problem to identify it, as far as no exact information copy would be capable to provide.

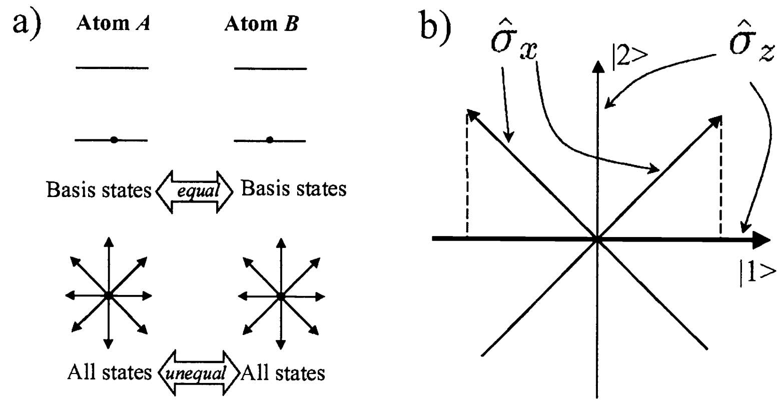

Figure 2. a) Equivalence of compatible basis-states ensembles and inequivalence of incompatible all-states ensembles of two two-level atoms. b) Vacuum fluctuations as a result of incompatibility: eigen states of $\hat{\sigma}_x$ have equal non-zero probabilities $p_\pm = 1/2$ at the eigen atomic state $|1\rangle$, thus providing nonzero fluctuations.

Let us, for example, discuss two atoms in the same state, shown in Fig. 2a.

Term "the same" yet demands refining in the case we are dealing actually with a quantum situation, by contrast with a classical one. In the latter case we take into account only the two basis states of each atom. Then we are free either to remember that they correspond to different atoms or suggest that it is one and the same atom, for there is actually only one non-zero probability state in a combined system. On contrary, it is far from that for quantum approach. Then atoms possess additional non-zero probability states, due to internal quantum uncertainty, as it is shown in Fig. 2b. This uncertainty, as it is well known, reveals for a harmonic oscillator in vacuum fluctuation energy $\hbar\omega/2$. In this case it takes the form of non-zero values $\hat{\sigma}_x^2 = \hat{\sigma}_y^2 = \hat{I}$, where Pauli matrices $\hat{\sigma}_{x,y}$ are treated as cosine and sine amplitudes of the atomic oscillator. The corresponding fluctuations differ for these two atoms, notwithstanding the latter are in the "same" state, that is it is not quite the same, as it belongs to different atoms possessing individual internal incompatible ensembles of quantum states. Indeed, the average differences $\left(\hat{\sigma}_x^A - \hat{\sigma}_x^B\right)^2$, $\left(\hat{\sigma}_y^A - \hat{\sigma}_y^B\right)^2$ are both different from zero, due to non-commutativity of their operators with the population operators $\hat{\sigma}_z$, the latter yield certainly zero difference $\hat{\sigma}_z^A - \hat{\sigma}_z^B$. To summarize, we can say, that when incompatible, that is non-orthogonal states of each atom (eigen states of the corresponding non-commuting operators) are involved, the states of two different atoms are ever different — with respect to all their ever coexisting internal quantum states, allowed by the quantum uncertainty.

One can express it in quantitative form as strictly positivity of the average operator of the square difference between the ortho-projectors onto the corresponding wave functions of the two atoms:

$$\hat{\varepsilon} = \int \Big(|\alpha\rangle\langle\alpha| \otimes \hat{I}_B - \hat{I}_A \otimes |\alpha\rangle\langle\alpha|\Big)^2 \frac{dV_\alpha}{D} = ||0\rangle\rangle\langle\langle 0|| + \frac{1}{3}\sum_{k=1}^{3} ||k\rangle\rangle\langle\langle k|| \geqq \frac{1}{3}.$$

Integration is made over the Bloch sphere of the states $|\alpha\rangle$ with the volume differential $dV_\alpha = \sin\vartheta\, d\vartheta d\varphi/(2\pi)$ and the total volume $V_\alpha = D = 2$. This bipartite operator has two eigen subspaces composed of a singlet and triplet Bell states $||k\rangle\rangle$ differing with the eigen square difference values $\varepsilon_k = 1, 1/3$, the singlet one is three times bigger.

From the above, the following conclusion can be drawn out. The determining term for the key difference between classical and quantum information is compatibility or incompatibility of the states involved with the information of interest. The one-time states of different systems are ever compatible. Due to this, they cannot copy one another if inside states of each system internally incompatible states are included. Vice versa, two-time states of the deterministically transformed system are ever incompatible. Two-time states of different systems may be other compatible or not. So the main classification of quantum information is to be based on its connection with the compatibility property. Up to now, four below described main types of information may be distinguished:

- *Classical* information — all the states are compatible and in original form of information theory no quantum systems are discussed.[5,6] It may be as well transmitted with quantum channels and also is of any interest in Quantum Physics.
- *Semiclassical* information — all the input information is given by classical states λ and the output states includes internal incompatibility in the form of all states of a Hilbert space H, but they are automatically compatible with the input states; the channel is generally described by classical parameter dependent ensemble of mixed states $\hat{\rho}_\lambda$.[10,11]
- *Coherent* information — both input and output are spaces composed of internally incompatible states, plus these spaces are also incompatible with one another and connected with a channel superoperator transforming the input density matrix to an output one: $\hat{\rho}_B = \mathcal{N}\hat{\rho}_A$.[12,13] It is a kind of flow of quantum incompatibility from one system to another.

Here we present a general discussion and some new results on the coherent information, and contribute the above list with an additional important type — *compatible* information. It is provided by a compound bipartite quantum system with the compatible input and output, which but include internal quantum incompatibility. Coherent and compatible information exhaust all possible qualitatively different types of information in quantum channels. Only compatible information turns consistent with the here presented discussion of information effectiveness of an experimental setup.

2. PHYSICAL MEANING OF COHERENT INFORMATION

Coherent information aims to quantitatively represent an amount of incompatible information which flu from one space to another, which actually may be one and the same space. A trivial case of coherent information exchange is dynamic evolution represented with a unitary time evolution operator, $\hat{\rho}_B = U\hat{\rho}_A U^{-1}$. Then all the pure states ψ allowed by the initial density matrix $\hat{\rho}_A$ are transformed with no distortion, and this way transmitted coherent information coincides with its initial amount. The latter is measured, by definition, with the von Neumann entropy, so

$$I_c = S[\hat{\rho}_B] = S[\hat{\rho}_A] = -\mathrm{Tr}\,\hat{\rho}_A \log \hat{\rho}_A. \tag{1}$$

This definition yet demands additional justifying in terms revealing an operational meaning of density matrix. Actually, density matrix appears in a self-consistent quantum theory as a result of averaging of some pure state in a compound system over the variables of no interest. Then Eq. 1 turns to be an entanglement of the input system A with some *reference* system R which yields a proper pure state Ψ_{AR}, $\mathrm{Tr}_R\,|\Psi_{AR}\rangle\langle\Psi_{AR}| = \hat{\rho}_A$ of a combined $A{+}R$ system. Thus quantitative measuring of coherent information is done in terms of mutually compatible states of two different systems, A and R, while information flows from input A to the output B, which differs from A here just with a unitary transformation.

To complete the general structure of information system an information channel $\mathcal{N}$ with the attached noisy *environment* E should be contributed, as it is shown in Fig. 3a matching the corresponding scheme of.[14]

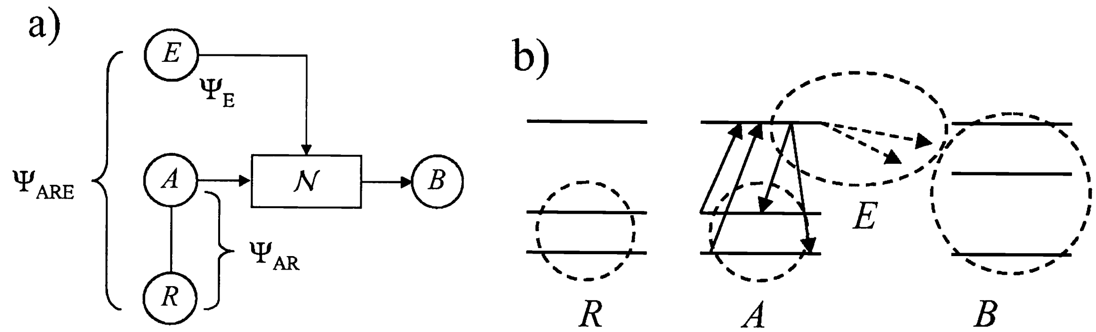

Figure 3. a) The most general scheme of quantum information system, composed of input A, reference system R, channel $\mathcal{N}$ with noisy environment E, and output B. b) An example of physical implementation of quantum information system: an input A and a reference system R are the ground states of two entangled atomic Λ-systems, information channel $\mathcal{N}$ is provided with laser excitation of an input system to the radiative upper level, the two photon field states corresponding to the emitted photons together with the vacuum state provide an output B, and all other field states together with the exited atomic state form an environment E.

The definition of coherent information for a general type channel is given[14] by

$$I_c = S[\hat{\rho}_B] - S\big[(\mathcal{N}\otimes\mathcal{I})\,|\Psi_{AR}\rangle\,\langle\Psi_{AR}|\big], \tag{2}$$

where $\mathcal{I}$ denotes the identical superoperator applied to the variables of the reference system. The second term is the *entropy exchange* which is non-zero due to exchange between the subsystems A+R and E, that is when $\mathcal{N} \neq \mathcal{I}$. Channel superoperator $\mathcal{N}$ transforms the states of input A according to the equation

$$\hat{\rho}_B = \mathcal{N}\hat{\rho}_A = \mathrm{Tr}_R(\mathcal{N}\otimes\mathcal{I})\,|\Psi_{AR}\rangle\,\langle\Psi_{AR}| \tag{3}$$

to the states of output B, which is again compatible with the reference system R because of no entanglement contributed between them at this transformation. A physical meaning of Eq. 2 is switched then from an incompatibility flow to a specific measure for a *preserved* entanglement between the *compatible* systems R and B, which is left after transmission through the channel. In a general case output B may be physically different from A and even represented with a Hilbert space of different structure, $H_B \neq H_A$,[15,16] as shown for a specific example of a physical information system in Fig. 3b.

What is the use of coherent information in physics? What quantum theory usually is applied to is calculation of some average values like $\langle\hat{A}\rangle = \sum \lambda_n \,\langle|n\rangle\langle n|\rangle$, where λ_n and $|n\rangle$ denote the eigen values and eigen vectors of operator $\hat{A}$. This expansion represents averaging of physical variables in terms of probabilities $P_n^A = \langle|n\rangle\langle n|\rangle$ of quantum states $|n\rangle$. As far as there is an innumerable set of all possible variables and it is much richer that the set of all quantum states, description of the correspondences between the physical states, apart of physical values, provides a more general information on the physical correspondences the most economical way. Laws of coherent information exchange follows the most basic lines of quantum physics, as they show the most general features of interaction between the two systems of interest chosen as input and output and connected with a one-to-one transformation of the input states. The dependencies of coherent information on the system parameters are more basic than the ones of specific physical values.

For example, as it was shown for a Dicke problem, an information exchange shows the same oscillation type of dynamics as the energetic exchange between the two atoms, assisted with the radiation damping.[15] Yet this oscillatory evolution is characteristic not only for energy but for the most other values, so it is absolutely reasonable to discuss right the evolution of the coherent information, keeping in mind its physical meaning as a preserved entanglement. The latter, in its turn, is a characteristic of an internal incompatibility exchange between to mutually compatible sets of states, H_R and H_A, corresponding to the reference and input systems. Among the other types of quantum information, coherent information distinguishes between the two types of information, corresponding to exchange via classical information and quantum entanglement. It is non-zero only for the latter case. So it is adequate to discuss how much a given information transmission channel preserves the capability of using the output as an equivalent of the input to realize a task, when specifically quantum properties of a signal are essential. A lot of literature is devoted to problems of this kind.[17]

One may also apply coherent information to a wider range of physical problems. For example, it may be rather interesting and contribute to a better understanding of a specific experiment, which is involved with some specific class of physical variables measured, to investigate the coherent information involved with the properly chosen model of a quantum channel. An investigation of this kind is presented in **Sec.** 4.

3. ONE-TIME COHERENT INFORMATION

The first approach to information characterization of a two-side quantum channel was a formal quantum generalization

$$I = S[\hat{\rho}_A] + S[\hat{\rho}_B] - S[\hat{\rho}_{AB}] \tag{4}$$

of the classical Shannon mutual information $I = S_A + S_B - S_{AB}$, if a joint density matrix $\hat{\rho}_{AB}$ is given and treated as a strict analogue of classical joint probability distribution P_{AB}.[18] Evidently, to apply this formula to quantum systems one should suggest that A and B states are mutually compatible, that is guaranteed for the one-time states of the corresponding physical systems, unless they are one and the same system discussed twice, both as input and output. Even on this account actual physical meaning of this quantity remains unclear.[19,20] It is no surprizing when taking into account the striking difference between the classical and quantum theories at discussion of physical channels. Generally, as it follows from Eq. (3), quantum input and output are incompatible, as we have for a single system at two time moments. So A and B cannot be treated as input and output, and their further specification should made for the quantum case.

Let us specify A as the reference system and B as the output for a given joint density matrix $\hat{\rho}_{AB}$, as it is shown in Fig. 4. The input B_0 and the channel $\mathcal{N}$ are not introduced explicitly but through their action, resulting in the given density matrix $\hat{\rho}_{AB}$.

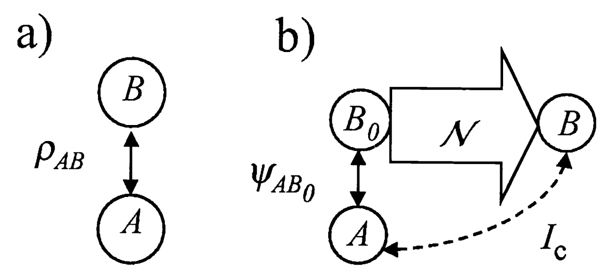

Figure 4: Reconstruction of a quantum information system corresponding to a given joint density matrix $\hat{\rho}_{AB}$.

The pure state Ψ_{AB_0} of the input–reference system and the channel superoperator $\mathcal{N}$ should obey the equation

$$\hat{\rho}_{AB} = \left(\mathcal{I}\otimes\mathcal{N}\right) |\Psi_{AB_0}\rangle\langle\Psi_{AB_0}| . \tag{5}$$

This automatically provides the coincidence of the partial density matrix of the reference state

$$\hat{\rho}_A = \mathrm{Tr}_{B_0} |\Psi_{AB_0}\rangle\langle\Psi_{AB_0}|$$

with the partial density matrix got by averaging of the given $A{+}B$ state: $\hat{\rho}_A = \mathrm{Tr}_B\hat{\rho}_{AB}$, as far as trace over B_0 of Eq. 5 is invariant on $\mathcal{N}$.

Then the corresponding *one-time* coherent information can be defined as

$$I_c = S[\hat{\rho}_B] - S[\hat{\rho}_{AB}], \tag{6}$$

that differs from the quantity (4) by lacking term $S[\hat{\rho}_A]$. Term "one-time" here in general case may not have a strict meaning, because any two compatible quantum systems A and B, even related to different time moments, can be treated as related to one time moment after the corresponding transformation of states.

Definition (6) is not symmetric, by contrast with (4). Moreover, as it is typical for coherent information, it may take negative values. The latter is evident for density matrices $\hat{\rho}_{AB}$ corresponding to purely classical information exchange via orthogonal bases, $\hat{\rho}_{AB} = \sum P_{ij} |i\rangle |j\rangle \langle j| \langle i|$. Then the entropies reduce to the classical entropies $S_{AB} = -\sum P_{ij} \log P_{ij}$, $S_B = -\sum P_j \log P_j$ and $S_{AB} > S_B$. Negative value of coherent information means that entropy exchange prevails information transmission, so it is reasonable to set $I_c = 0$ in this case.

4. COHERENT INFORMATION EXCHANGE RATE WITH Λ-SYSTEM

Information system presented in Fig. 3 is of special interest for modern investigations in the field of new applications based on nonclassical properties of quantum information, like quantum cryptography and quantum computations. It is based on atomic Λ-systems, which are suggested to be a promising type to store quantum information in a stable form of the ground state qubit and convenient to manipulate with laser radiation.[21] Treating another Λ-system as a reference one has a reasonable justification, as the entanglement of two corresponding qubits is a clear physical meaning of the initially provided quantum information. Discussion of the radiation channel is interesting, because the transformation of initial qubit into the photon field enables a wide choice of subsequent transformations. One of the questions involved is, in particular, how rapidly could the information be recycled after a single use of a qubit–photon field channel?

Basic calculations of coherent information for this channel were done in.[16] In particular, its dependence on time and laser field action angle for a symmetric Λ-system is shown in Fig. 5a for a maximal entropy qubit state $\hat{\rho}_A = \hat{I}/2$, when information does not depend on the individual field intensities of the two applied laser field.

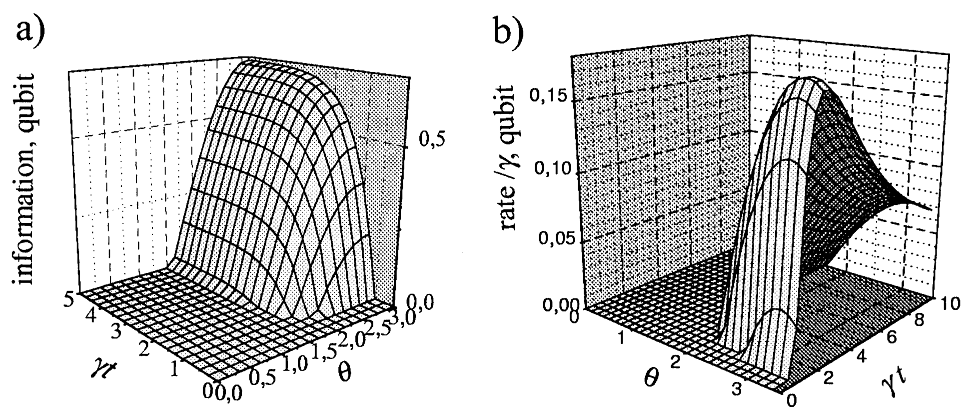

Figure 5. a) Coherent information in a symmetric Λ-system as a function of dimensionless time γt and action angle $\theta = \Omega\tau_p$ for the maximum entropy input state; γ is the decay rate, Ω is the effective Rabi frequency and τ_p is the exciting pulse duration.[15] b) Dependence of the information rate on the cycle duration t and action angle $\theta = \Omega\tau_p$.

It is easy to infer from the graph of Fig. 5a that there is an optimum for information rate $R = I_c/t$, $t = \tau_c$, if we introduce a periodic use of the information channel with a cycle duration τ_c, so that after each cycle the initial state is instantaneously renewed. The calculations results for rate R for a symmetric Λ-system with the partial decay rates $\gamma_1 = \gamma_2 = \gamma$ is shown in Fig. 5b.[22] The total optimum rate is $R_0 = 0.178\gamma$. Thus the process of atom–photon field information exchange sets the corresponding rate limit on using of the coherent information stored in Λ-systems. The order of its magnitude is given by the decay rate of the excited state, while an exact value depends on the partial decay rates $\gamma_{1,2}$ of the Λ-system transitions. At the limit of a two-level radiative system, $\gamma_1 = 0$ or $\gamma_2 = 0$, the optimum rate is equal to 0.316γ.

5. COMPATIBLE INFORMATION

When interested only in one-time average values, one can restrict representation of quantum internal incompatibility in an equivalent form of classical probability distribution on the quantum states of interest. Indeed, the probability measure

$$P(d\alpha) = \langle\alpha|\,\hat{\rho}_A\,|\alpha\rangle\, dV_\alpha \tag{7}$$

on the space of all quantum states given, the average value of any operator $\hat{A} = \sum \lambda_n\,|n\rangle\,\langle n|$ can be represented in the form $\langle\hat{A}\rangle = \sum \lambda_n dP/dV_\alpha(\alpha_n)$, where $|\alpha_n\rangle = |n\rangle$. Here dV_α, $\int dV_\alpha = D$ denotes the volume differential in the space of physically different states of the D-dimensional Hilbert space H_A, that is, for example, is the Bloch sphere for a qubit system with $D = 2$ (see **Sec.** 2). Eq. (7) is an average of the projective measure

$$\hat{E}(d\alpha) = |\alpha\rangle\,\langle\alpha|\, dV_\alpha, \tag{8}$$

which is a specific case of non-orthogonal expansion of unit,[23] or Positive Operator-Valued Measure (POVM).[24] POVMs represent some physical measurement procedures made in a compound space $H_A \otimes H_a$ with an appropriate additional space H_a and joint density matrix $\hat{\rho}_A \otimes \hat{\rho}_a$, which implies no additional information on A beyond the one given by $\hat{\rho}_A$.

Let us assume that two Hilbert spaces, H_A and H_B, of the corresponding quantum systems A and B, and the joint density matrix $\hat{\rho}_{AB}$ in $H_A \otimes H_B$, are given. They may correspond, in particular, to the subsystems of a compound system $A+B$, given at the same time instant t, or some other way defined input and output of an abstract quantum channel of some real physical system. Thus described subsystems A and B are compatible and let to introduce a joint measurement represented with two POVMs in the form $\hat{E}_A \otimes \hat{E}_B$, which provides no correlations between output and input measurements. Then we get the responding joint input–output probability distribution

$$P(d\alpha, d\beta) = \mathrm{Tr}\left[\hat{E}_A(d\alpha) \otimes \hat{E}_B(d\beta)\right] \hat{\rho}_{AB}. \tag{9}$$

The corresponding Shannon information $I = S[P(d\alpha)] + S[P(d\beta)] - S[P(d\alpha, d\beta)]$ defines then a ***compatible*** information measure.[25]

Depending on the specific choice of measurement the physical meaning of compatible information is representation of the quantum information on input obtainable from the output via the POVMs, which select the information of interest in the classical form of the corresponding α and β variables the information carriers. An important case when α and β enumerate all the quantum states of H_A and H_B, in accordance with Eq. (8), is worth to be distinguished. In this case compatible information is distributed over all quantum states and involved with internal quantum uncertainty, which is taken into account in the distribution (7), and specifically quantum correlations due to possible entanglement between A and B are taken into account in the joint probability (9). Moreover, in this case compatible information yields *operational invariance* property,[26] that is all the non-commuting physical variables are taken into account an equivalent way. This kind of classical representation of quantum information may be related to the ones arising at using representations of Quantum Mechanics in terms of classical variables, like Wigner or Glauber ones.[27]

6. INFORMATION AMOUNT FROM AN EXPERIMENTAL SETUP

The above discussion of generalized measurements encourages to introduce a likelihood mathematical concept of information attainable by an experimental setup, which certainly is one of the key goals of Quantum Information theory. The main problem is to provide a general structure of the corresponding information system. To resolve it, one has to specify the information of interest, which is actually the most difficult point. Two possible solutions are proposed here and illustrated by the block scheme of Fig. 6.

This scheme corresponds to a typical mathematical structure of a density matrix of a complex system, including two transformations, $\mathcal{A}$ and $\mathcal{B}$, representing control and measurement interactions, correspondingly:

$$\hat{\rho}_{\mathrm{out}} = \mathcal{B}\mathcal{N}\mathcal{A}\,\hat{\rho}_{\mathrm{in}}. \tag{10}$$

Here $\hat{\rho}_{\mathrm{in}}$ and $\hat{\rho}_{\mathrm{out}}$ are the initial and final density matrices for the collection of the degrees of freedom, chosen in a mathematical model of the setup. Superoperator $\mathcal{A}$ is associated with the preparation of the information,

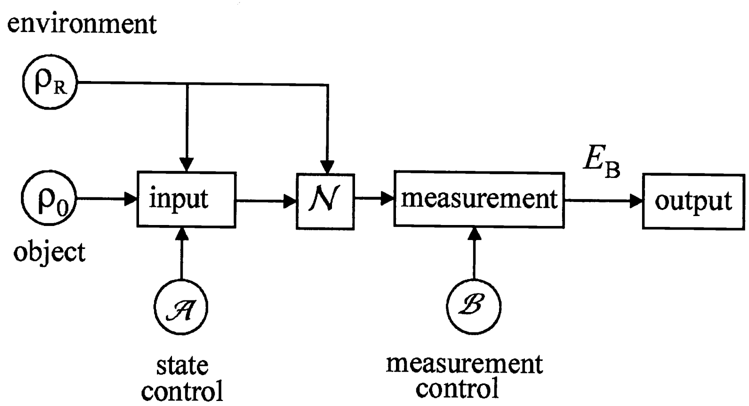

Figure 6. Information structure of a quantum experimental setup. An object accompanied with noise environment undergoes the state control interactions, produces the input information ensemble, depending on either of object dynamical parameters or quantum states of interest, then after some channel superoperator transformation $\mathcal{N}$ the output information is measured. $\mathcal{A}$ and $\mathcal{B}$ denote transformations provided with the controlling interactions, E_B is a particular representation of a measurement procedure in the form of the corresponding POVM.

$\mathcal{B}$ with the measurement and $\mathcal{N}$ with the transmission of the information to the output, that is an internal information channel. This markovian-type structure is not the most general one, as for simplicity it implies that the reservoirs corresponding to each transformation are independent and their density matrices can be separated from $\hat{\rho}_{\text{in}}$. Only under this restriction we get a separated combination of the three superoperators and the input density matrix, so we have to remember that a proper generalization of Eq. (10) may be demanded in a general case. The above simplification makes possible to get a relatively simple mathematical representation of the information structure in terms of corresponding decompositions of $\mathcal{A}$ and $\mathcal{B}$.

Preparation of information ever implies applying some interactions, resulting in the corresponding transformations, which are unitary if only all the involved degrees of freedom are taken into account. Generally, it includes also an interaction with a reservoir, so it is represented with a non-unitary superoperator. We provide a sketch of it for two possible choices of a physical information of interest:
a) the system dynamic parameters a,
b) the system dynamic states $|a\rangle$.

If the information goal choice is "a)", it is available via the dynamical evolution operators $U_A(a)$, which in its turn may depend either on controlling parameters c. *A ' priory* information on a is included in a proper chosen probability measure $\mu(da)$. Corresponding superoperator $\mathcal{A}$ is then given as $\mathcal{A} = \int \mathcal{A}_a \mu(da)$ with

$$\mathcal{A}_a = \langle U_A(a) \odot U_A^{-1}(a) \rangle_E \tag{11}$$

where symbol $\odot$ denotes the place to substitute a density matrix transformed and $\langle\rangle_E$ denotes averaging over the noise environment.

If the information goal choice is "b)", it is available via the measurement superoperator transformation composed of superoperators

$$\mathcal{A}_a = \langle |a\rangle \langle a| \odot |a\rangle \langle a| \rangle_E \tag{12}$$

as the corresponding sum $\mathcal{A} = \sum \mathcal{A}_a$ is a measurement superoperator represented as an averaged standard decomposition $\sum_i \hat{A}_i \odot \hat{A}_i^+$ of a completely positive trace-preserving superoperator,[28] with a properly specified operators $\hat{A}_i = \hat{A}_i^+ \to |a\rangle \langle a|$. Keeping in mind, that a may represent a continuous variable, we have to use a generalized representation $\mathcal{A} = \int \mathcal{A}_a \mu(da)$ in the integration form with a proper measure $\mu(da)$, providing a corresponding decomposition of unit (POVM) $\int |a\rangle \langle a| \mu(da) = \hat{I}$.

A most general concept of superoperator sets like (11), (12) is represented with an arbitrary *Positive Superoperator Measure* (PSM) $\mathcal{A}(da) = \mathcal{A}_a \mu(da)$, which is a decomposition of a completely positive trace-preserving superoperator, alike POVM $\hat{E}(da)$ is a positive decomposition of a unit operator. It satisfies the conditions of complete positivity, $\mathcal{A}(da)\hat{\rho} \geqq 0$, and normalization, $\mathrm{Tr}\int\mathcal{A}(da)\hat{\rho} = 1$. The latter may be expressed in an equivalent form of preservation of the unit operator $\int \mathcal{A}^*(da)\hat{I} = \hat{I}$ by the conjugate PSM $\mathcal{A}^*$.

Once more a special case when the POVMs are represented by Eq. (8) with all the states of the corresponding to $\mathcal{A}$ and $\mathcal{B}$ Hilbert spaces H_A, H_B is worth to point out. This specification assigns an information restrictions not as much to an entire setup as to the basic physical limitations underlying the chosen mechanism to obtain quantum information. Here the latter is represented in a "solid" classical form enabling its copying and free use, that may as well be assigned "by silence" to the meaning of the term "information", by contrast to the opposing meaning of coherent information discussed in **Sec.** 2–4.

Repeating the above argumentation for a measurement superoperator $\mathcal{B}=\int\mathcal{B}(db)=\int\mathcal{B}_b\nu(db)$ with $\mathcal{B}_b$ of the same form Eq. (12), we get an implementation of the input and output information in the form of classical variables a and b, for the both "a)" and "b)" choices of the information of interest. The corresponding joint probability distribution is then given by

$$P(da, db) = \mathrm{Tr}\,\mathcal{B}(db)\mathcal{N}\mathcal{A}(da)\,\hat{\rho}_{\mathrm{in}}. \tag{13}$$

This distribution is ever positive and normalized to 1. It provides an experimenter with the statistical correspondence between the states of interest and output information available with the setup. The corresponding "informativity" of the setup can be expressed then in the quantitative form as the responding Shannon information. It then may be used for optimization of the setup parameters.

It is important to mark, that mutual compatibility of the $|a\rangle$ and $|b\rangle$ states (for "a)" choice) is not declared here, and in general case they may correspond to non-commuting projectors. As a trivial extreme, they may be the same states, and all the information is sent with zero error probability. If they belong to different physical subsystems, they may carry on quantum correlations due to the corresponding structure of the channel superoperator $\mathcal{N}$. A simplest example is given by $\mathcal{N} = U_{AB} \odot U_{AB}^{-1}$ with U_{AB} an entangling unitary transformation.

The control parameters c may be either fixed or discussed as a set of used values $c \in \mathbb{C}$. In any case using Shannon information measure, an optimization of c or set $\mathbb{C}$ gets available. As for the unknown probability distribution $\mu(da)$ of the dynamical parameters a of the case "a)", this problem is no concern with quantum mechanics and well understood in terms of classical decision theory.[29] As for the specification of the action $\mathcal{B}_b$ of the measurement system in the form (11), it may be generalized in the form of a general type PSM. Two PSMs $\mathcal{A}(da)$ and $\mathcal{B}(db)$ cover a very wide range of state control and measurement systems incorporated into setup.

7. CONCLUSIONS

From the above, the following conclusions may be drawn out.

Classical, semiclassical, coherent and *compatible* information exhaust basically different types of quantum information.

Physical meaning of coherent information is an amount of the internal incompatibility exchanged between two systems and measured as an entanglement *preserved* between the output and the reference system.

Here presented *one-time* coherent information sets a correct correspondence between the Schumacher's and modified Stratonovich's approaches.

Coherent information exchange *rate* of a Λ-system via photon field cannot exceed 0.178γ for a symmetric system and 0.316γ for a general case.

Compatible information is and adequate characteristic of a quantum information exchange between compatible systems, available in terms of classical information despite internal incompatibility, in contrast to coherent information, which is basically irreducible to classical terms.

Internal compatibility of the input and output quantum information seems an adequate restriction for a physical information flow in an experimental setup. It makes possible quantitative characterization of the available *information effectiveness*. Then information exchange between the subsystem preparing information and the measuring device is formulated as a probabilistic correspondence between the classical variables determining the corresponding dynamical evolution, and the measured output values. A general mathematical representation of information generation and its readout is suggested in the form of two Positive Superoperator Measures. This representation of physical information exchange in an experimental setup promises to make Quantum Information theory directly applicable to the demands of experimental physics.

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REFERENCES

1. A. Sudbery, *Quantum Mechanics and the Particles of Nature*, Cambridge Univ. Press, New York, 1986.
2. C. A. Fuchs, LANL e-print quant-ph/0205039 (2002).
3. I. Devetak and A. E. Staples, LANL e-print quant-ph/0112166 (2001).
4. R. B. Griffiths, Phys. Rev. A 66, 012311 (2002).
5. C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication* (University of Illinois Press, Urbana, 1949).
6. R. G. Gallagher, *Information Theory and Reliable Communication* (John Wiley and Sons, New York, 1968).
7. Č. Brukner, A. Zeilinger, LANL e-print quant-ph/0006087.
8. M. J. W. Hall, LANL e-print quant-ph/0007116.
9. Č. Brukner, A. Zeilinger, LANL e-print quant-ph/0008091.
10. A. S. Holevo, Probl. Inf. Trans. **9**, 177 (1973).
11. M. J. W. Hall, Phys. Rev. **A 55**, 100 (1997).
12. B. Schumacher and M. A. Nielsen, Phys. Rev. **A 54**, 2629 (1996).
13. S. Lloyd, Phys. Rev. **A 55**, 1613 (1997).
14. H. Barnum, B. W. Schumacher, and M. A. Nielsen, Phys. Rev. **A 57**, 4153 (1998).
15. B. A. Grishanin and V. N. Zadkov, Phys. Rev. **A 62**, 032303 (2000).
16. B. A. Grishanin and V. N. Zadkov, Laser Physics **10**, No. 6, 1280 (2000).
17. Bouwmeester D, Ekert A, Zeilinger A (Eds.) *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, Berlin: Springer, 2000.
18. R. L. Stratonovich, Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika **8**, 116 (1965).
19. G. Lindblad, Lect. Notes Phys. **378**, *Quantum Aspects of Optical Communication*, ed. by C. Benjaballah, O. Hirota, S. Reynaud, 71 (1991).
20. A. S. Holevo, LANL e-print quant-ph/9809022.
21. I. V. Bargatin, B. A. Grishanin and V. N. Zadkov, Physics–Uspekhi **171**, 625 (2001).
22. D. Bokarev, 4-course student presentation, Chair of Gen. Phys. and Wave Processes, Faculty of Physics, Moscow State University (2001).
23. B. A. Grishanin, Tekhnicheskaya Kibernetika, **11** (5), 127 (1973).
24. J. Preskill, *Lecture notes on Physics 229: Quantum information and computation*, available in Internet: http://www.theory.caltech.edu/people/preskill/ph229/.
25. B. A. Grishanin and V. N. Zadkov, Laser Physics **11**, No. 12, 1324 (2001).
26. C. Brukner and A. Zeilinger, Phys. Rev. Lett. **83**, 3354 (1999).
27. R. J. Glauber, "Optical coherence and photon statistics", in: C. DeWitt, A. Blandin, C. Cohen-Tannoudji (Eds), *Quantum Optics and Electronics*, Gordon&Breach, New York–London–Paris, 1965.
28. K. Kraus, *States, Effects and Operations*, Springer Verlag, Berlin, 1983.
29. A. Wald, *Statistical Decision Functions*, Wiley, New York, 1950.